



Implementing the Common Core State Standards for Mathematics: The CCSS Curriculum Materials Analysis Tools

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FYI

Presentation slides will be posted on the
NCSM website:

mathedleadership.org

or

email me at

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What is NCSM?

International organization of and for mathematics education leaders:

Coaches and mentors

Curriculum leaders

Department chairs

District supervisors/leaders

Mathematics consultants

Mathematics supervisors

Principals

Professional developers

Publishers and authors

Specialists and coordinators

State and provincial directors

Superintendents

Teachers

Teacher educators

Teacher leaders

mathedleadership.org



Today's Goals

- Familiarize you with the “major advances” of the CCSS
- Introduce you to the *CCSS Curriculum Materials Analysis Tools*
- Identify other resources to support implementation of CCSS

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NCSM
NATIONAL COUNCIL OF
SUPERVISORS



LEADERSHIP IN MATHEMATICS EDUCATION
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43rd NCSM Annual Conference
Indianapolis, Indiana • April 11–13, 2011

On Track for Student Success: Mathematics Leadership

Fuel your leadership engine

Be there for the green flag as Karen Cator's keynote address, *Transforming American Education: Learning Powered by Technology* opens the 43rd NCSM Annual Conference on April 11, 2011.

Karen Cator is the Director of the Office of Educational Technology at the U.S. Department of Education. She has devoted her career to creating the best possible learning environments for the current generation of students. Prior to joining the department, Cator directed Apple's leadership and advocacy efforts in education. In this role, she focused on the intersection of education policy and research, emerging technologies, and the reality faced by teachers, students, and administrators.

Prior to joining Apple in 1997, Cator worked in the public education sector leading technology planning and implementation in Juneau, Alaska. She also served as Special Assistant for Telecommunications for the Lieutenant Governor of Alaska. Cator holds a master's degree in school administration from the University of Oregon and a bachelor's in early

Karen
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Make your plans and save the dates:
Together, let's race to the top!

NEWSLETTER

The National Council of Supervisors of Mathematics Improving Student Achievement Series

No. 7/Spring 2010

Research-Informed Answers for Mathematics Education Leaders

LEADERSHIP IN MATHEMATICS EDUCATION
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Improving Student Achievement in Mathematics by Promoting Positive Self-Beliefs

... many students have difficulty in school not because they are incapable of performing successfully but because they are incapable of believing they can perform successfully ... Consequently, parents and teachers do well to take seriously their share of the responsibility in nurturing the self-beliefs of their children and students, for it is clear that these beliefs can have beneficial or destructive influences.

Pajares & Schunk, 2002

Our Position

The National Council of Supervisors of Mathematics believes that in order to help students learn challenging, standards-based mathematics, educators must establish a classroom climate that promotes positive self-beliefs about intelligence and academic ability. We believe that teacher actions can significantly affect students' self-beliefs and that — as these student self-beliefs deepen and strengthen — teacher beliefs do so as well. Positive self-beliefs, as well as positive experiences in mathematics, increase student motivation and engagement.

Mathematics educators can best instill positive student beliefs about their intelligence and ability to do mathematics when we:

- Understand that educators play a crucial role in student motivation.
- Know that equity requires that educators reflect on their individual beliefs about intelligence and whether or not they believe that all children can learn mathematics.
- Establish a learning environment that promotes a view of intelligence as malleable and fosters a sense of belonging for each student.
- Recognize and act upon the fact that even students who currently appear not to care, do want to learn and be challenged.
- Ensure that all students have the right to authentic and meaningful mathematics curricula taught in engaging and accessible ways.

- Use mathematics as a forum for students to reach a better understanding of themselves as learners by providing opportunities for them to experience and recognize that hard work and perseverance results in deeper understanding and higher achievement.
- Teach and model the meaning of effective effort.
- Foster positive and encouraging relationships with students and among students by providing opportunities for students to engage in peer-to-peer learning communities.
- Implement assessment for learning strategies that involve students in goal setting, presentations of their learning, and self-reflections.
- Provide descriptive feedback to students about their work to help students identify the strengths and weaknesses of their mathematics strategies and suggest action steps for improvement.

Research that Supports Our Position

In its *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics (2000) put forth an ambitious vision of school mathematics that requires that all students engage in meaningful mathematics. For students even to try engaging in meaningful mathematics, however, it is critical that we not underestimate what it takes to motivate them to succeed in school. The National Mathematics Advisory Panel (2008), for example, found that 62% of Algebra I teachers reported "working with unmotivated students" is the "single most challenging aspect of teaching Algebra I successfully." In addition, former American Psychological Association president Robert Stemborg

The NCSM Improving Student Achievement Series is a set of position papers designed to provide research-based practices for school and district mathematics education leaders.

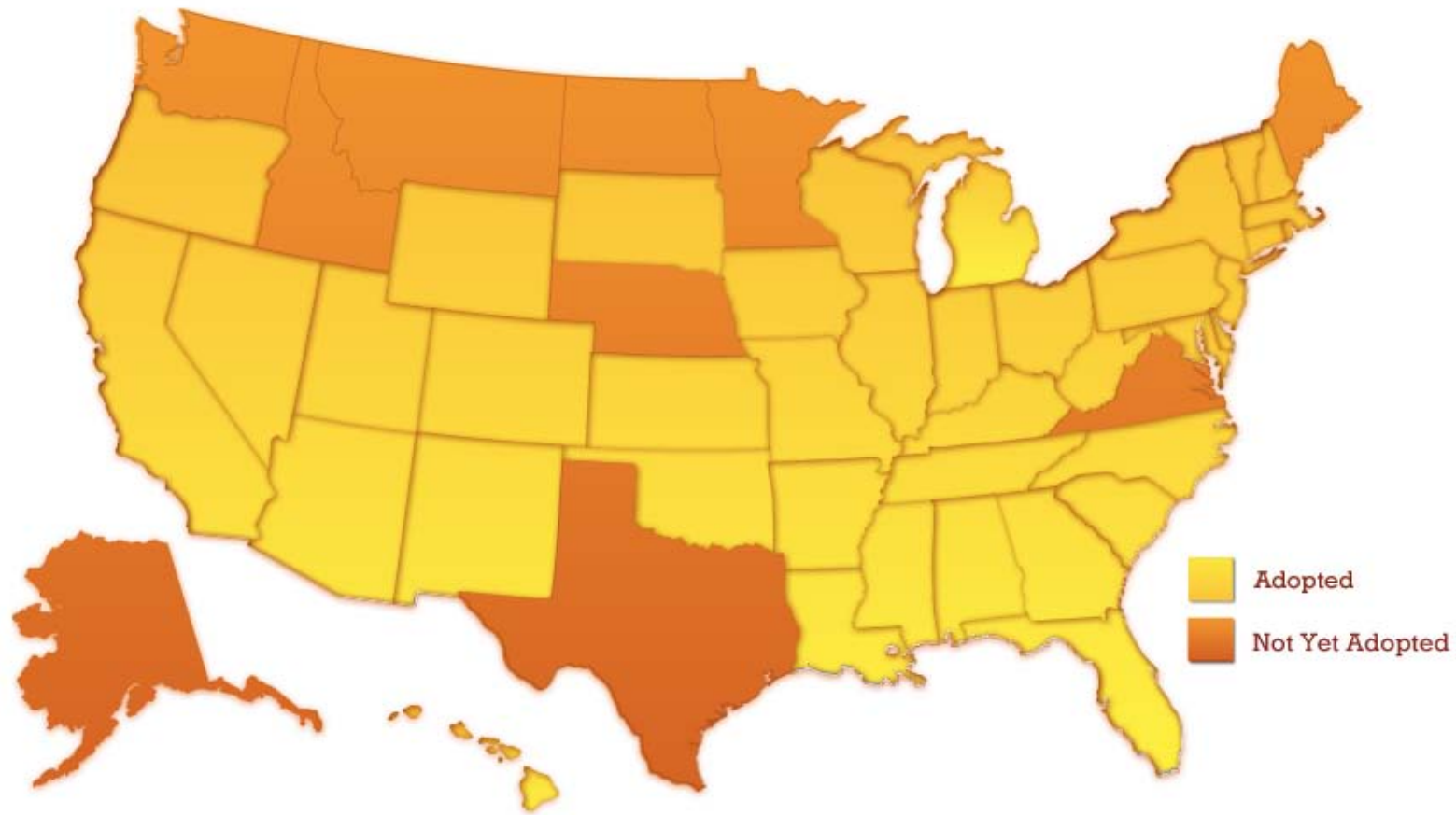


NCSM Position Papers

1. Effective and Collaborative Teams
2. Sustained Professional Learning
3. Equity
4. Students with Special Needs
5. Assessment
6. English Language Learners
7. Positive Self-Beliefs
8. Technology

mathedleadership.org

“The Common Core State Standards represent an opportunity – once in a lifetime – to form effective coalitions for change.” Jere Confrey, August 2010





CCSS: A Major Challenge/Opportunity

- College and career readiness expectations
- Rigorous content and applications
- Stress conceptual understanding as well as procedural skills
- Organized around mathematical principles
- Focus and coherence
- Designed around research-based learning progressions whenever possible.



Common Core State Standards for Mathematics

- Introduction
 - Standards-Setting Criteria
 - Standards-Setting Considerations
- Application of CCSS for ELLs
- Application to Students with Disabilities
- Mathematics Standards
 - Standards for Mathematical Practice
 - Contents Standards: K-8; HS Domains
- Appendix A: Model Pathways for High School Courses

COMMON CORE STATE STANDARDS FOR

Mathematics

Expanded Version*



*Contains the Overall Standards Introduction, Standards-setting Criteria, Standards-setting Considerations, Applications of the Standards for English Language Learners (ELLs) and Students with Disabilities, and the Mathematics Standards.

Expanded CCSS and Model Pathways available at
www.mathedleadership.org/



What's different about CCSS?

- Accountability
- Accountability
- Accountability



What's different about CCSS?

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

— CCSS (2010, p.5)



Assessment Consortia

- Partnership for the Assessment of Readiness for College and Careers (PARCC)
<http://www.fldoe.org/parcc/>
- SMARTER Balanced Assessment Consortium
<http://www.k12.wa.us/SMARTER/>



Implementing CCSS

- Challenge:
 - CCSS assessments not available for several years (2014-2015 deadline).
- Where NOT to start--
 - Aligning CCSS standards grade-by-grade with existing mathematics standards.



Implementing CCSS: Where to Start?

- Build understanding of:
 - Standards for Mathematical Practice
 - Content Standards Progressions: Domains and Clusters
- Assess implications of this for current practice
 - Short-term changes
 - Long-term changes



Implementing CCSS: Where to Start?

- Mathematical practices
- Progressions within and among content clusters and domains
- “Key advances”
- Conceptual understanding
- Research-Informed C-T-L-A Actions
- Assessment tasks
 - Balanced Assessment Tasks (BAM)
 - State released tasks



Collaborate!

Engage teachers in working in collaborative teams

- Grade level/course/department meetings
 - Common assessments
 - Common unit planning
 - Differentiating instruction
- Cross grade/course meetings
 - End-of-year/Beginning-of-year expectations



Common Core State Standards for Mathematics

- Introduction
 - Standards-Setting Criteria
 - Standards-Setting Considerations
- Application of CCSS for ELLs
- Application to Students with Disabilities
- Mathematics Standards
 - Standards for Mathematical Practice
 - Standards for Mathematical Content: K-8; HS Domains
- Appendix A: Model Pathways for High School Courses

Standards for Mathematical Practice

“The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.”

(CCSS, 2010)

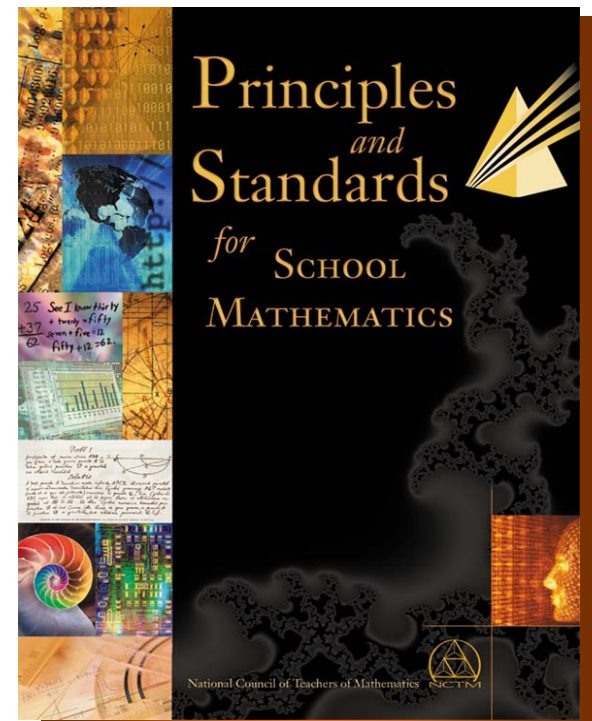


Underlying Frameworks

National Council of Teachers of Mathematics

5 **Process** Standards

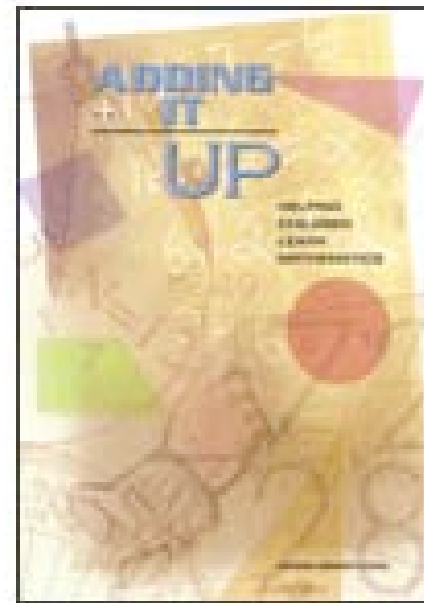
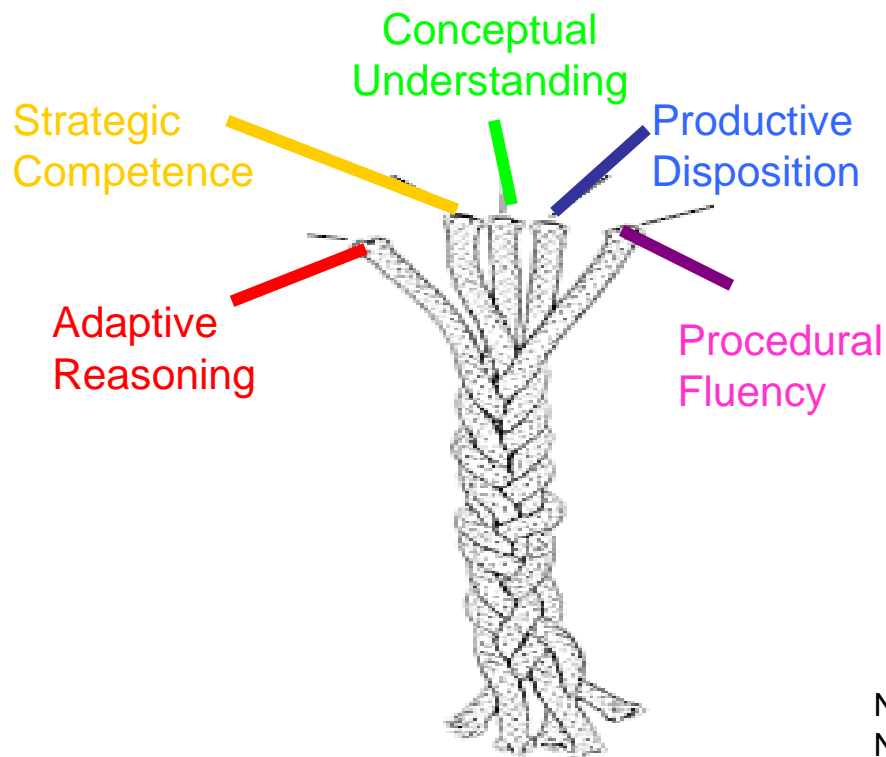
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations



NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

Underlying Frameworks

Strands of Mathematical Proficiency



NRC (2001). *Adding It Up*. Washington, D.C.: National Academies Press.



Strands of Mathematical Proficiency

- ***Conceptual Understanding*** – comprehension of mathematical concepts, operations, and relations
- ***Procedural Fluency*** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic Competence*** – ability to formulate, represent, and solve mathematical problems
- ***Adaptive Reasoning*** – capacity for logical thought, reflection, explanation, and justification
- ***Productive Disposition*** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.



Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



The Standards for Mathematical Practice

Take a moment to examine the first three words of each of the 8 mathematical practices... what do you notice?

Mathematically Proficient Students...



The Standards for [Student] Mathematical Practice

What are the *verbs* that illustrate the student actions for your assigned mathematical practice?

Circle, highlight or underline them for your assigned practice...

Discuss with a partner:

What jumps out at you?



The Standards for [Student] Mathematical Practice

SMP1: *Explain and make conjectures...*

SMP2: *Decontextualize and contextualize...*

SMP3: *Understand and use...*

SMP4: *Apply and interpret...*

SMP5: *Consider and detect...*

SMP6: *Communicate precisely to others...*

SMP7: *Discern and recognize...*

SMP8: *Notice and pay attention to...*



The Standards for [Student] Mathematical Practice

*On a scale of 1 (low) to 6 (high),
to what extent is your school/district promoting
students' proficiency in the practice you
discussed?*

Evidence for your rating?

Individual rating

Team rating



Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
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Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Reasoning and explaining

Modeling and using tools

Seeing structure and generalizing

Standards for Mathematical Practice in a Classroom

McDonald's Claim



Wikipedia reports that 8% of all Americans eat at McDonalds every day.

310 million Americans and 12,800 McDonalds...

Do you believe the Wikipedia report to be true?

Create a mathematical argument to
justify your position.



McDonald's Claim Problem

- Individually
- Discuss with a partner
- With people at your table
 - Discuss and try to reach consensus
 - Be prepared to present your argument to the group



McDonald's Claim Problem

Reflecting on other's arguments

- Similar to yours
- Different from yours
- How does their argument influence your thinking about your argument?



McDonald's Claim Problem

- Which mathematical *practices* are needed to complete the task?
- What mathematics *content* is needed to complete the task?



Standards for Mathematical Practice in a Classroom

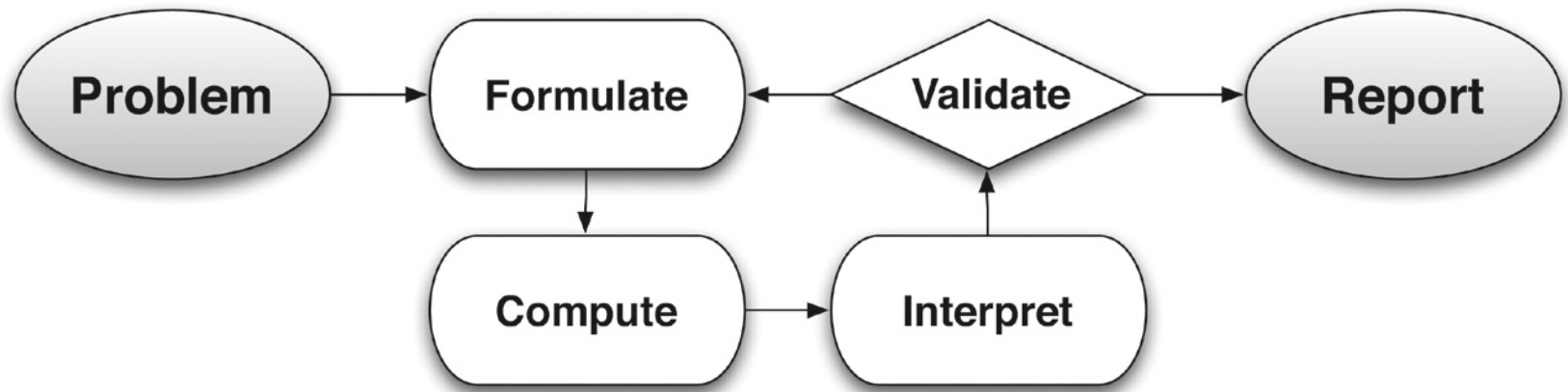
MP 3. Construct viable arguments and critique the reasoning of others

- Students make conjectures
- Students justify their conclusions and communicate them to others
- Students compare the effectiveness of two plausible arguments
- Students listen and respond to the arguments of others for sense making and clarity

MP 4: Model with Mathematics

- Making assumptions and approximations to simplify a complicated situation
- Identify important quantities in a practical situation
- Analyze relationships mathematically to draw conclusions
- Interpret their mathematical results in the context of the situation
- Reflect on whether the results make sense.

Modeling Cycle





Standards for Mathematical Practice in a Classroom

HS N-Q: Reason quantitatively and use units to solve problems

- N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems;
- N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards for Mathematical Practice

- Describe the thinking processes, habits of mind and dispositions that students need to develop a deep, flexible, and enduring understanding of mathematics; in this sense they are also a means to an end.

SP1. Make sense of problems

“....they [students] analyze givens, constraints, relationships and goals.they monitor and evaluate their progress and change course if necessary. and they continually ask themselves “Does this make sense?”



Standards for Mathematical Practice

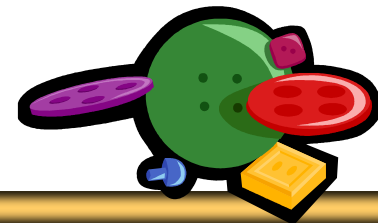
AND....

- Describe mathematical content students need to learn.

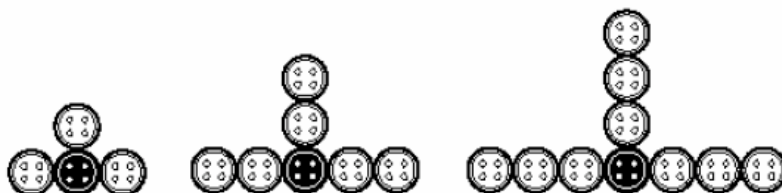
SP1. Make sense of problems

“..... students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.”

Buttons Task



Gita plays with her grandmother's collection of black & white buttons. She arranges them in patterns. Her first 3 patterns are shown below.



Pattern #1

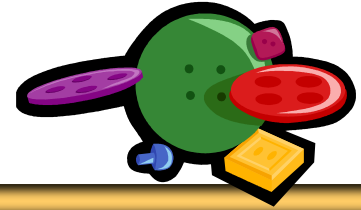
Pattern #2

Pattern #3

Pattern #4

1. Draw pattern 4 next to pattern 3.
2. How many white buttons does Gita need for Pattern 5 and Pattern 6? Explain how you figured this out.
3. How many buttons in all does Gita need to make Pattern 11? Explain how you figured this out.
4. Gita thinks she needs 69 buttons in all to make Pattern 24. How do you know that she is **not** correct?
How many buttons does she need to make Pattern 24?

Button Task



1. Individually complete parts 1 - 3.
2. Then work with a partner to compare your work. Look for as many ways to solve part 3 as possible.
3. Which mathematical *practices* are needed to complete the task?

www.Inside Mathematics.org



A reengagement
lesson using the
Button Task

Francis Dickinson
San Carlos Elementary
Grade 5

<http://www.insidemathematics.org/index.php/classroom-video-visits/public-lessons-numerical-patterning/218-numerical-patterning-lesson-planning?phpMyAdmin=NqJS1x3gaJqDM-1-8LXtX3WJ4e8>

Learner A

3. How many buttons in all does Gita need to make Pattern 11?

34 buttons

Explain how you figured this out.

$$1 \times (11 \times 3) + 1 = 34 \text{ buttons}$$

I added one for the black button
in the middle

Pictorial Representation

What does Learner A see staying the same? What does Learner A see changing?
Draw a picture to show how Learner A sees this pattern growing through the first 3 stages. Color coding and modeling with square tiles may come in handy.

Verbal Representation

Describe in your own words how Learner A sees this pattern growing. Be sure to mention what is staying the same and what is changing.

Learner B

3. How many buttons in all does Gita need to make Pattern 11?

$4 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$ 34 ✓
Explain how you figured this out.

I added $4 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$
 $= 34$ which is the # of buttons

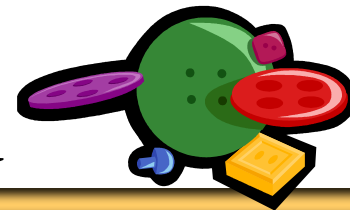
Pictorial Representation

What does Learner B see staying the same? What does Learner B see changing?
Draw a picture to show how Learner B sees this pattern growing through the first 3 stages. Color coding and modeling with square tiles may come in handy.


Verbal Representation

Describe in your own words how Learner B sees this pattern growing. Be sure to mention what is staying the same and what is changing.

Button Task Revisited



- Which of the Standards of Mathematical Practice did you see the students working with?
- What did Mr. Dickinson get out of using the same math task two days in a row, rather than switching to a different task(s)?
- How did the way the lesson was facilitated support the development of the Standards of Practice for students?
- What implications for implementing CCSS does this activity suggest to you?



Standards for [*Student*] Mathematical Practice

“Not all tasks are created equal, and **different tasks will provoke different levels and kinds of student thinking.**”

Stein, Smith, Henningsen, & Silver, 2000

“The level and kind of thinking in which students engage determines what they will learn.”

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997



The Standards for [Student] Mathematical Practice

The 8 Standards for Mathematical Practice –
place an emphasis on student demonstrations
of learning...

Equity begins with an understanding of how
the selection of tasks, the assessment of
tasks, the student learning environment
creates great inequity in our schools...



Why Focus on Mathematical Tasks?

- Tasks form the basis for students' opportunities to learn what mathematics is and how one does it;
- Tasks influence learners by directing their attention to particular aspects of content and by specifying ways to process information;
- The level and kind of thinking required by mathematical instructional tasks influences what students learn; and
- Differences in the level and kind of thinking of tasks used by different teachers, schools, and districts, is a major source of inequity in students' opportunities to learn mathematics.



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Comparing Two Mathematical Tasks

Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

Stein, Smith, Henningsen, & Silver, 2000, p. 1



Comparing Two Mathematical Tasks

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits.

1. If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?
2. How long would each of the sides of the pen be if they had only 16 feet of fencing?
3. How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.



Cognitive Level of Tasks

- Lower-Level Tasks
(e.g., Martha's Carpeting Task)
- Higher-Level Tasks
(e.g., The Fencing Task)

The Quasar Project

Briars, October 2011



Lower-Level Tasks

- Memorization
 - What are the decimal equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?
- Procedures without connections
 - Convert the fraction $\frac{3}{8}$ to a decimal.



Higher-Level Tasks

- Procedures with connections
 - Using a 10 x 10 grid, identify the decimal and percent equivalents of $\frac{3}{5}$.
- Doing mathematics
 - Shade 6 small squares in a 4 x 10 rectangle. Using the rectangle, explain how to determine:
 - a) The decimal part of area that is shaded;
 - b) The fractional part of area that is shaded.



Implementation Issue

Do *all* students have the opportunity to engage in mathematical tasks that promote students' attainment of the mathematical practices on a regular basis?



Opportunities for *all* students to engage in challenging tasks?

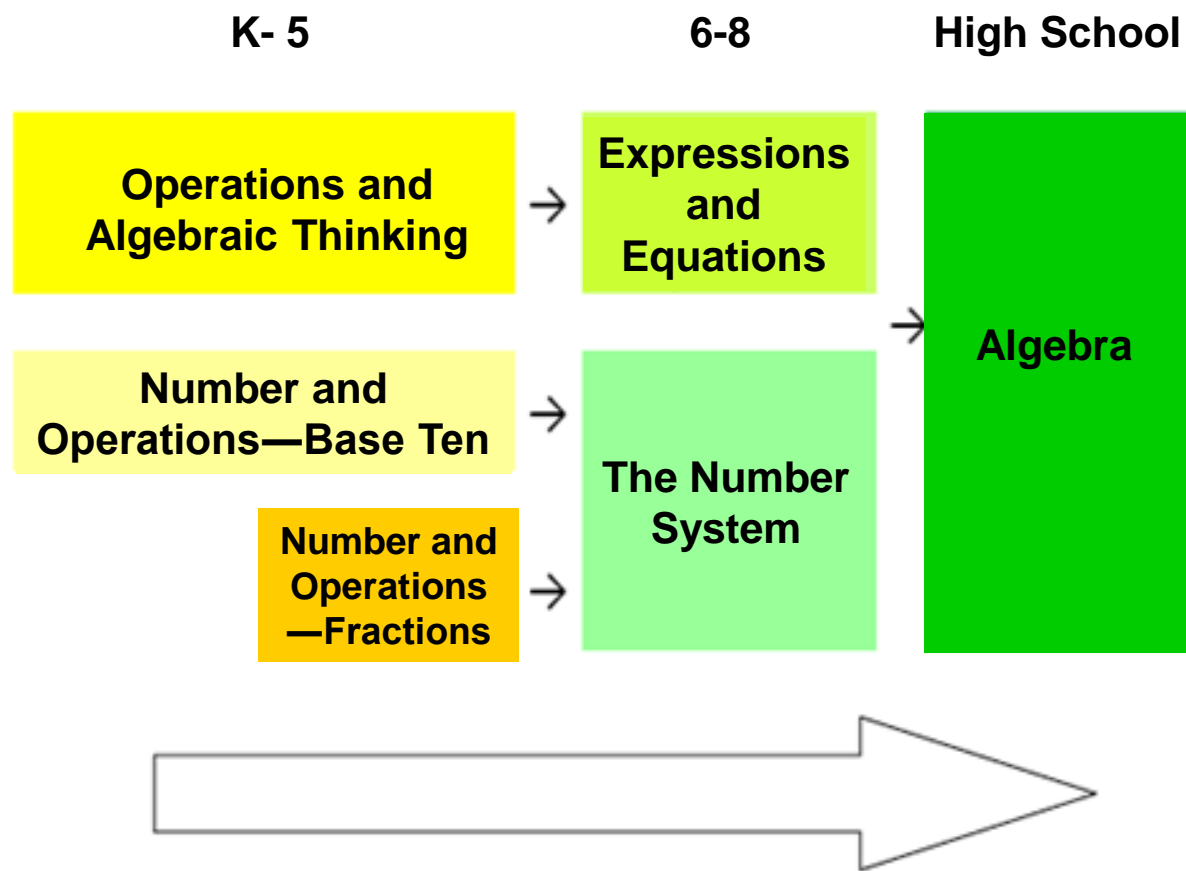
- Examine tasks in your instructional materials:
 - Higher cognitive demand?
 - Lower cognitive demand?
- Where are the challenging tasks?
- Do *all* students have the opportunity to grapple with challenging tasks?
- Examine the tasks in your assessments:
 - Higher cognitive demand?
 - Lower cognitive demand?



Standards for Mathematical Content

- Counting and Cardinality (K)
- Operations and Algebraic Thinking (K-5)
- Number and Operations in Base Ten (K-5)
- Measurement and Data (K-5)
- Geometry (K-HS)
- Number and Operations — Fractions (3-5)
- Ratios and Proportional Relationships (6-7)
- The Number System (6-8)
- Expressions and Equations (6-8)
- Statistics and Probability (6-HS)
- Functions (8-HS)
- Number and Quantity (HS)
- Algebra (HS)
- Modeling (HS)

Progressions within and across Domains



Daro, 2010



Standards for Mathematical Content

- Counting and Cardinality (K)
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- Functions (8-HS)
- Number and Quantity (HS)
- Algebra (HS)
- Modeling (HS)

Functions Overview	Algebra Overview
<p><i>Interpreting Functions</i></p> <ul style="list-style-type: none"> – Understand the concept of function and use function notation – Interpret functions that arise in applications in terms of the context – Analyze functions using different representations <p><i>Building Functions</i></p> <ul style="list-style-type: none"> – Build a function that describes a relationship between two quantities – Build new functions from existing functions <p><i>Linear, quadratic and exponential models</i></p> <ul style="list-style-type: none"> – Construct and compare linear, quadratics, and exponential models and solve problems – Interpret expressions for functions in terms of the situation they model <p><i>Trigonometric Functions</i></p> <ul style="list-style-type: none"> – Extend the domain of the trigonometric functions using the unit circle – Model periodic phenomena using trigonometric functions – Prove and apply trigonometric identities 	<p><i>Seeing Structure in Expressions</i></p> <ul style="list-style-type: none"> – Interpret the structure of expressions – Write expressions in equivalent forms to solve problems <p><i>Arithmetic with Polynomials and Rational Expressions</i></p> <ul style="list-style-type: none"> – Perform arithmetic operations on polynomials – Understand the relationship between zeros and factors of polynomials – Use polynomial identities to solve problems – Rewrite rational expressions <p><i>Creating Equations</i></p> <ul style="list-style-type: none"> – Create equations that describe numbers or relationships <p><i>Reasoning with Equations and Inequalities</i></p> <ul style="list-style-type: none"> – Understand solving equations as a process of reasoning and explain the reasoning – Solve equations and inequalities in one variable – Solve systems of equations – Represent and solve systems of equations and inequalities graphically



High School Standards

Interpreting Functions

F-IF

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
 - a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
 - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
 - d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Model Course Pathways

Courses in higher level mathematics: Precalculus, Calculus (upon completion of Precalculus), Advanced Statistics, Discrete Mathematics, Advanced Quantitative Reasoning, or other courses to be designed at a later date, such as additional career technical courses.

Algebra II

Geometry

Algebra I

Pathway A

Traditional in U.S.

Mathematics
III

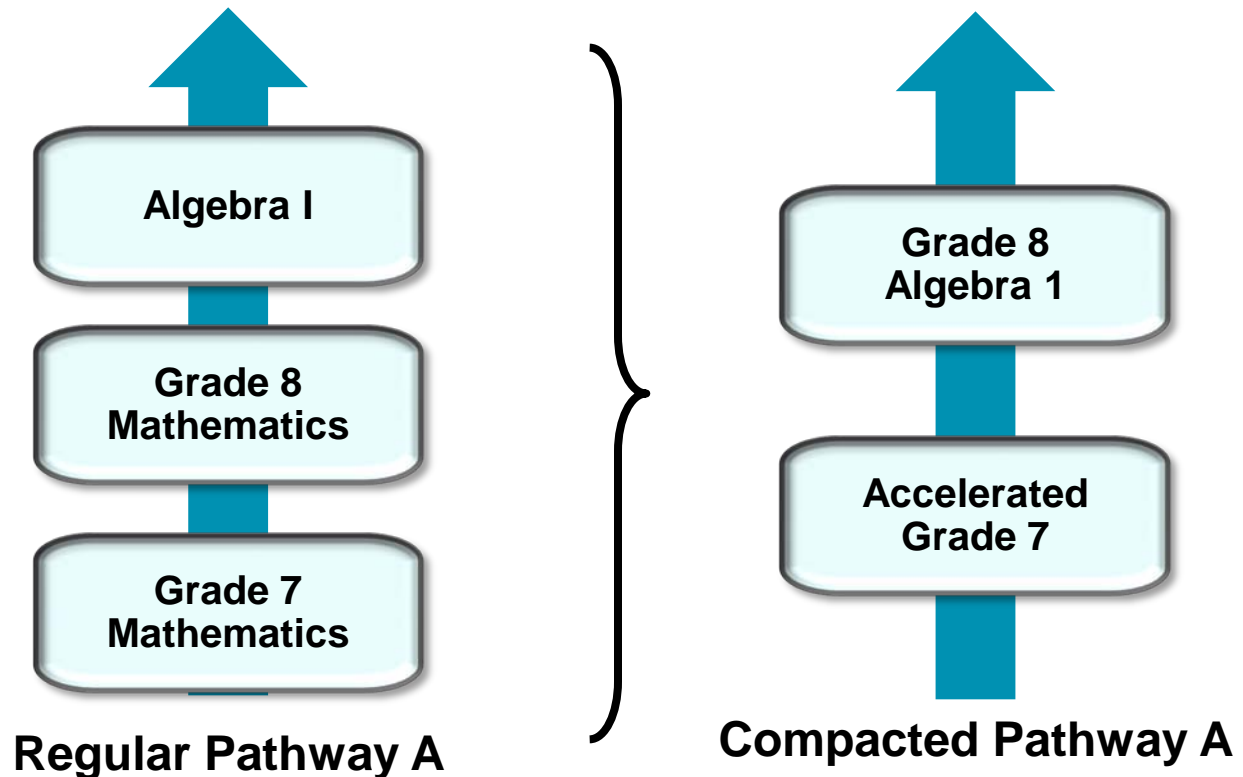
Mathematics II

Mathematics I

Pathway B

*International Integrated approach (typical
outside of U.S.)*

Compacted Course Pathways





Key Advances

1. Operations and the problems they solve
2. Properties of operations: Their role in arithmetic and algebra
3. Mental math and “algebra” vs. algorithms
4. Units and unitizing
 - a. Unit fractions
 - b. Unit rates
5. Defining congruence and similarity in terms of transformations
6. Quantities-variables-functions-modeling
7. Number-expression-equation-function
8. Modeling

Daro, 2010



Write a word problem that could be modeled by


Elementary

$$a \times b = p$$

Middle/High


$$y = rx$$

where r is a rate



Word problems for $a \times b = p$ or $y = rx$

- Product (p) or result (y) unknown?
 - There are 3 bags with 6 plums in each bag? How many plums are there in all?
 - A car is traveling at an average speed of 60 miles/hr. How far will it travel in 2.5 hours?
- Group size (b) or x unknown?
 - If 18 plums are shared equally among 3 bags, how many plums will be in each bag?
 - A car is traveling at an average speed of 60 miles/hr. How long will it take to travel 150 miles?



Word problems for $a \times b = p$ or $y = rx$

- Number of groups (a) or rate (r) unknown?
 - If 18 plums are to be packed 6 to a bag, then how many bags are needed?
 - A car traveled 150 miles in 2 hours. What was its average speed?
- Contexts:
 - Equal groups? Arrays? Area? Size comparison?
 - Unit rates? Non-unit rates? Continuous quantities? Discrete quantities?

Common Multiplication and Division Situations

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,⁴ Area⁵	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$



	Operations and Algebraic Thinking	Numbers and Operations in Base Ten	Fractions
1	Understand and apply properties of operations and the relationship between addition and subtraction.	Use place value understanding and properties of operations to add and subtract.	
2		Use place value understanding and properties of operations to add and subtract.	
3	Understand properties of multiplication and the relationship between multiplication and division.	Use place value understanding and properties of operations to perform multi-digit arithmetic. <i>A range of algorithms may be used.</i>	
4		Use place value understanding and properties of operations to perform multi-digit arithmetic. <i>Fluently add and subtract multi-digit whole numbers using the standard algorithm.</i>	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
5		Perform operations with multi-digit whole numbers and with decimals to hundredths. <i>Fluently multiply multi-digit whole numbers using the standard algorithm.</i>	Apply and extend previous understandings of multiplication and division to multiply and divide fractions.



If 2 pounds of beans cost \$5, how much
will 15 pounds of beans cost?

CCSS de-emphasizes means/extremes as
solution method

$$\frac{2}{5} = \frac{15}{X}$$

$$2x = 5 \cdot 15$$



If 2 pounds of beans cost \$5, how much
will 15 pounds of beans cost?

Method 1

pounds	2	4	6	8	10	12	14	1	15
dollars	5	10	15	20	25	30	35	2.50	37.50

CCSS Ratio & Proportional Relationships Progression, 9/2011

If 2 pounds of beans cost \$5, how much will 15 pounds of beans cost?

Method 2

		$\xrightarrow{+2}$		$\xrightarrow{+15}$		
pounds		2		1		15
<hr/>						
dollars		5		2.50		37.50
		$\xrightarrow{+2}$		$\xrightarrow{+15}$		

CCSS Ratio & Proportional Relationships Progression, 9/2011

If 2 pounds of beans cost \$5, how much will 15 pounds of beans cost?

Method 3

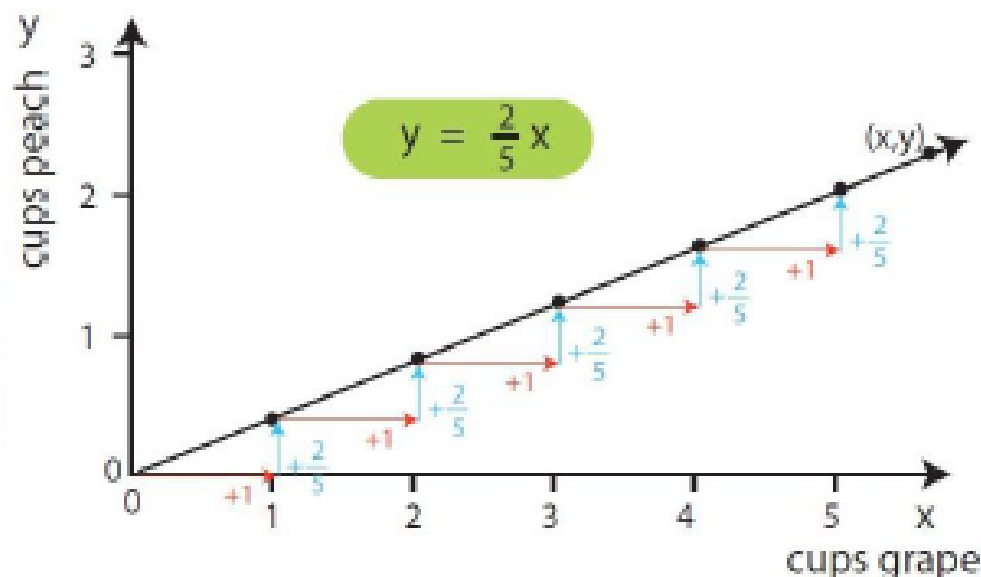
	$\cdot \frac{15}{2}$	
pounds	2	15
dollars	5	37.50
	$\cdot \frac{15}{2}$	

CCSS Ratio & Proportional Relationships Progression, 9/2011

Correspondence among a table, graph, and equation of a proportional relationship

For every 5 cups grape juice, mix in 2 cups peach juice.

x cups grape	y cups peach
(0)	(0)
5	2
1	$\frac{2}{5}$
2	$2 \cdot \frac{2}{5}$
3	$3 \cdot \frac{2}{5}$
4	$4 \cdot \frac{2}{5}$
x	$x \cdot \frac{2}{5}$



On the graph: For each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

When you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.

When you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.

When you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.

When you go x units to the right, you go up $x \cdot \frac{2}{5}$ units.

Starting from (0, 0), to get to a point (x, y) on the graph, go x units to the right, so go up $x \cdot \frac{2}{5}$ units.

Therefore $y = x \cdot \frac{2}{5}$

$$y = \frac{2}{5}x$$



Key Advances

1. Operations and the problems they solve
2. Properties of operations: Their role in arithmetic and algebra
3. Mental math and “algebra” vs. algorithms
4. Units and unitizing
 - a. Unit fractions
 - b. Unit rates
5. Defining congruence and similarity in terms of transformations
6. Quantities-variables-functions-modeling
7. Number-expression-equation-function
8. Modeling

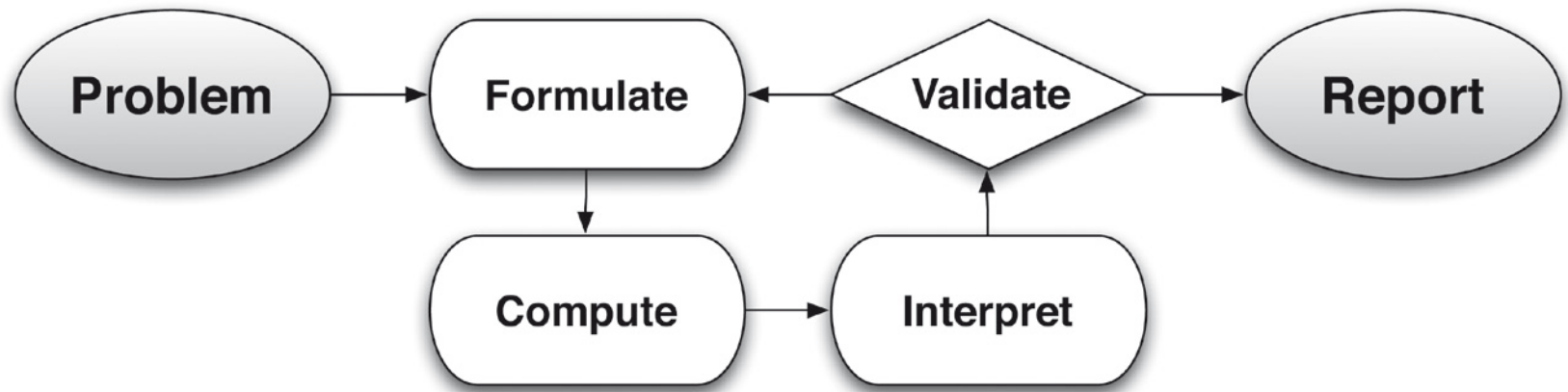
Daro, 2010




Modeling

- Practice K-12
- Content Conceptual Category HS
Standards embedded in other categories (★)
- Examples of models:
 - Equations: Writing total cost as a product of unit price and number bought
 - Geometric shape to represent physical object

Modeling Cycle





Examples of Situations to be Modeled

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.



Key Advances

1. Operations and the problems they solve
2. Properties of operations: Their role in arithmetic and algebra
3. Mental math and “algebra” vs. algorithms
4. Units and unitizing
 - a. Unit fractions
 - b. Unit rates
5. Defining congruence and similarity in terms of transformations
6. Quantities-variables-functions-modeling
7. Number-expression-equation-function
8. Modeling

Daro, 2010



Please sit in grade-band groups

- Elementary
- Middle
- High School



CCSS ANALYZING CURRICULUM MATERIALS TOOLS



CCSS Curriculum Materials Analysis Tools

- Key question: To what extent will these materials support faithful implementation of CCSS?
- Provide assistance in collecting useful information focused on salient issues related to the CCSSM, to ensure consistency across reviewers, and promote discussions about mathematics curriculum materials.



Development Team

- William S. Bush (chair), Mathematics Educator, University of Louisville, Kentucky
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Financial Support for the Curriculum Analysis Tools

- Brookhill Foundation (Kathy Stumpf)
- Texas Instruments (through CCSSO)



Tool Development Process

- Development Team formed in October 2010
- First version of tools developed in November 2010
- Initial drafts of tools piloted with groups of elementary middle, and high school teachers in December 2010
- Tools revised based on these pilots
- Tools reviewed by postsecondary mathematics educators, mathematicians, and public school administrators nationally in January 2011
- Tools revised based on input from these reviewers to obtain final versions in April 2011



Analysis Tool Components

- Content Analysis Tool
- Mathematical Practices Analysis Tool
- Overarching Considerations
 - Equity
 - Assessment
 - Technology



Analysis Tool Components

- **Content Analysis Tool**
- Mathematical Practices Analysis Tool
- Overarching Considerations
 - Equity
 - Assessment
 - Technology



Content Analysis Tool

- In-depth analysis of core content
- Examine content progressions across grades/courses



K-5 Mathematics Content

- Operations and Algebraic Thinking, Grades K-2
- Operations and Algebraic Thinking, Grades 3-5
- Number and Operations in Base Ten, Grades K-2
- Number and Operations in Base Ten, Grades 3-5
- Number and Operation –Fractions, Grades 3-5
- Geometry, Grades K-2
- Geometry, Grades 3-5



6-8 Mathematics Content

- Ratio and Proportional Relationships
- Expressions and Equations
- Statistics and Probability
- Geometry



9-12 Mathematics Content

- Interpreting Functions
- Reasoning with Equations and Inequalities
- Similarity, Right Triangles and Trigonometry
- Geometric Measurement and Dimension
- Interpreting Categorical and Quantitative Data

CCSSM Curriculum Analysis Tool 1—Operations and Algebraic Thinking for Grades K-2

Name of Reviewer _____ School/District _____ Date _____

Name of Curriculum Materials _____ Publication Date _____ Grade Level(s) _____

Content Coverage Rubric (Cont): Not Found (N) -The mathematics content was not found. Low (L) - Major gaps in the mathematics content were found. Marginal (M) -Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled. Acceptable (A)-Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled. High (H)-The content was fully formed as described in the standards.	Balance of Mathematical Understanding and Procedural Skills Rubric (Bal): Not Found (N) -The content was not found. Low (L)-The content was not developed or developed superficially. Marginal (M)-The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills. Acceptable (A)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed. High (H) - The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.
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CCSSM Grade K				CCSSM Grade 1				CCSSM Grade 2			
K.OA Operations and Algebraic Thinking	Chap. Pages	Cont N-L-M-A-H	Bal N-L-M-A-H	1.OA Operations and Algebraic Thinking	Chap. Pages	Cont N-L-M-A-H	Bal N-L-M-A-H	2.OA Operations and Algebraic Thinking	Chap. Pages	Cont N-L-M-A-H	Bal N-L-M-A-H
Understand addition as putting together and adding to, and subtraction as taking apart and taking from				Represent and solve problems involving addition and subtraction				Represent and solve problems involving addition and subtraction			
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.				1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions e.g., by using objects, drawings, and equations with a symbol for the unknown number. <i>Common addition and subtraction situations. Adding To or Taking From situations with result unknown, change unknown, and start unknown. Put Together/ Take Apart with total unknown, added unknown or both addends unknown.</i> 2. Solve word problems that call for addition of three whole				1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 1 Add and subtract within 20. 3. Determine whether a group of objects (up to 20) has an odd or even number of members. Write an equation to express the total as a sum of equal			

CCSSM Curriculum Analysis Tool 1—Similarity, Right Triangles, and Trigonometry & Trigonometric Functions in Grades 9-12

Name of Reviewer _____ School/District _____ Date _____

Name of Curriculum Materials _____ Publication Date _____ Course(s) _____

Content Coverage Rubric (Cont):

Not Found (N) -The mathematics content was not found.

Low (L) - Major gaps in the mathematics content were found.

Marginal (M) -Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled.

Acceptable (A)-Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled.

High (H)-The content was fully formed as described in the standards.

Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):

Not Found (N) -The content was not found.

Low (L)-The content was not developed or developed superficially.

Marginal (M)-The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills.

Acceptable (A)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed.

High (H)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.

CCSSM Standards Grades 9-12	Chapter pages	Cont N-L-M-A-H	Bal N-L-M-A-H	Notes/Explanation
Similarity, Right Triangles, and Trigonometry (G-SRT)				
Understand similarity in terms of similarity transformations				
1. Verify experimentally the properties of dilations given by a center and a scale factor:				
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.				
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.				
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.				
3. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding				



Content Analysis Tool Rubric

Content Coverage Rubric (Cont):

- Not Found (N) - The mathematics content was not found.
- Low (L) - Major gaps in the mathematics content were found.
- Marginal (M) - Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled.
- Acceptable (A) - Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled.
- High (H) - The content was fully formed as described in the standards



Content Analysis Tool Rubric

Balance of Mathematical Understanding and Procedural Skills Rubric (Bal):

- Not Found (N) - The content was not found.
- Low (L) - The content was not developed or developed superficially.
- Marginal (M) - The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills.
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- High (H)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.



Content Summary Discussion

Overall Impressions:

1. What are your overall impressions of the curriculum materials examined?
2. What are the strengths and weaknesses of the materials you examined?



Content Summary Discussion

Standards Alignment:

3. Have you identified gaps within this domain? What are they? If so, can these gaps be realistically addressed through supplementation?
4. Within grade levels, do the curriculum materials provide sufficient experiences to support student learning within this standard?
5. Within this domain, is the treatment of the content across grade levels consistent with the progression within the Standards?



Content Summary Discussion

Balance between Mathematical Understanding and Procedural Skills

6. Do the curriculum materials support the development of students' mathematical understanding?
7. Do the curriculum materials support the development of students' proficiency with procedural skills?
8. Do the curriculum materials assist students in building connections between mathematical understanding and procedural skills?
9. To what extent do the curriculum materials provide a balanced focus on mathematical understanding and procedural skills?
10. Do student activities build on each other within and across grades in a logical way that supports mathematical understanding and procedural skills?



Try Tool 1

- Select one domain
- Examine the extent to which your textbook supports faithful implementation of this domain.



Analysis Tool Components

- Content Analysis Tool
- **Mathematical Practices Analysis Tool**
- Overarching Considerations
 - Equity
 - Assessment
 - Technology

CCSSM Curriculum Analysis Tool 1—Operations and Algebraic Thinking for Grades K-2

Name of Reviewer _____ School/District _____ Date _____

Name of Curriculum Materials _____ Publication Date _____ Grade Level(s) _____

Content Coverage Rubric (Cont): Not Found (N) -The mathematics content was not found. Low (L) - Major gaps in the mathematics content were found. Marginal (M) -Gaps in the content, as described in the Standards, were found and these gaps may not be easily filled. Acceptable (A)-Few gaps in the content, as described in the Standards, were found and these gaps may be easily filled. High (H)-The content was fully formed as described in the standards.	Balance of Mathematical Understanding and Procedural Skills Rubric (Bal): Not Found (N) -The content was not found. Low (L)-The content was not developed or developed superficially. Marginal (M)-The content was found and focused primarily on procedural skills and minimally on mathematical understanding, or ignored procedural skills. Acceptable (A)-The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, but the connections between the two were not developed. High (H) - The content was developed with a balance of mathematical understanding and procedural skills consistent with the Standards, and the connections between the two were developed.
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CCSSM Grade K				CCSSM Grade 1				CCSSM Grade 2			
K.OA Operations and Algebraic Thinking	Chap. Pages	Cont N-L-M-A-H	Bal N-L-M-A-H	1.OA Operations and Algebraic Thinking	Chap. Pages	Cont N-L-M-A-H	Bal N-L-M-A-H	2.OA Operations and Algebraic Thinking	Chap. Pages	Cont N-L-M-A-H	Bal N-L-M-A-H
Understand addition as putting together and adding to, and subtraction as taking apart and taking from				Represent and solve problems involving addition and subtraction				Represent and solve problems involving addition and subtraction			
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.				1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions e.g., by using objects, drawings, and equations with a symbol for the unknown number. <i>Common addition and subtraction situations. Adding To or Taking From situations with result unknown, change unknown, and start unknown. Put Together/ Take Apart with total unknown, added unknown or both addends unknown.</i> 2. Solve word problems that call for addition of three whole				1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem 1 Add and subtract within 20. 3. Determine whether a group of objects (up to 20) has an odd or even number of members. Write an equation to express the total as a sum of equal			

Name of Reviewer _____ School/District _____ Date _____

Name of Curriculum Materials _____ Publication Date _____ Grade Level(s) _____

Tool 1 Domain Considered _____

**Opportunities to Engage in the Standards for Mathematical Practices
Found Across the Content Standards**

Overarching Habits of Mind	1. Make sense of problems and persevere in solving them.	6. Attend to precision.
Evidence of how the Standards for Mathematics Practice were addressed (with page numbers)		
Reasoning and Explaining	2. Reason abstractly and quantitatively.	3. Construct viable arguments and critique the reasoning of others.
Evidence of how the Standards for Mathematics Practice were addressed (with page numbers)		

(Mathematical Practices → Content) To what extent do the materials demand that students engage in the Standards for Mathematical Practice as the primary vehicle for learning the Content Standards?

(Content → Mathematical Practices) To what extent do the materials provide opportunities for students to develop the Standards for Mathematical Practice as “habits of mind” (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards?

To what extent do accompanying assessments of student learning (such as homework, observation checklists, portfolio recommendations, extended tasks, tests, and quizzes) provide evidence regarding students’ proficiency with respect to the Standards for Mathematical Practice?

What is the quality of the instructional support for students’ development of the Standards for Mathematical Practice as habits of mind?

Summative Assessment

(Low) – The Standards for Mathematical Practice are not addressed or are addressed superficially.

(Marginal) The Standards for Mathematical Practice are addressed, but not consistently in a way that is embedded in the development of the Content Standards.

(Acceptable) – Attention to the Standards for Mathematical Practice is embedded throughout the curriculum materials in ways that may help students to develop them as habits of mind.

Explanation for score



Mathematical Practices Analysis Tool Rubric

Mathematical Practices → Content

- To what extent do the materials demand that students engage in the Standards for Mathematical Practice as the primary vehicle for learning the Content Standards?

Content → Mathematical Practices

- To what extent do the materials provide opportunities for students to develop the Standards for Mathematical Practice as “habits of mind” (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards?



Mathematical Practices Analysis Tool Rubric

Assessment

- To what extent do accompanying assessments of student learning (such as homework, observation checklists, portfolio recommendations, extended tasks, tests, and quizzes) provide evidence regarding students' proficiency with respect to the Standards for Mathematical Practice?

Support

- What is the quality of the instructional support for students' development of the Standards for Mathematical Practice as habits of mind?



Try Tool 2

- Use the shaded standards in the content domain you analyzed with Tool 1.
- Examine the extent to which your textbook supports students' acquisition of the mathematical practices.



Analysis Tool Components

- Content Analysis Tool
- Mathematical Practices Analysis Tool
- Overarching Considerations
 - Equity
 - Assessment
 - Technology



Overarching Concerns - Equity

To what extent do the materials:

1. Provide teachers with strategies for meeting the needs of a range of learners?
2. Provide instructional support to help teachers sequence or scaffold lessons so that students move from what they know to what they do not know?
3. Provide opportunities for teachers to use a variety of grouping strategies?
4. Embed tasks with multiple entry-points that can be solved using a variety of solution strategies or representations?
5. Suggest accommodations and modifications for English language learners that will support their regular and active participation in learning mathematics?



Overarching Concerns - Equity

To what extent do the materials:

6. Provide opportunities to use reading, writing, and speaking in mathematics lessons.
7. Encourage teachers to draw upon home language and culture to facilitate learning?
8. Encourage teachers to draw on multiple resources such as objects, drawings, and graphs to facilitate learning?
9. Draw upon students' personal experiences to facilitate learning?
10. Provide opportunities for teacher and students to connect mathematics to other subject areas?



Overarching Concerns - Equity

To what extent do the materials:

11. Provide both individual and collective opportunities for students to learn using mathematical tasks with a range of challenge?
12. Provide opportunities for advanced students to investigate mathematics content at greater depth?
13. Provide a balanced portrayal of various demographic and personal characteristics?



Overarching Concerns - Assessment

To what extent do the materials:

14. Provide strategies for gathering information about students' prior knowledge and background?
15. Provide strategies for teachers to identify common student errors and misconceptions?
16. Assess students at a variety of knowledge levels (e.g., memorization, understanding, reasoning, problem solving)?
17. Encourage students to monitor their own progress?
18. Provide opportunities for ongoing review and practice with feedback related to learning concepts, and skills.
19. Provide support for a varied system of on-going formative and summative assessment (formal or informal observations, interviews, surveys, performance assessments, target problems)?



Overarching Concerns - Technology

To what extent do the materials:

20. Integrate technology such as interactive tools, virtual manipulatives/objects, and dynamic mathematics software in ways that engage students in the Mathematical Practices?
21. Include or reference technology that provides opportunities for teachers and/or students to communicate with each other (e.g. websites, discussion groups, webinars)?
22. Include opportunities to assess student mathematical understandings and knowledge of procedural skills using technology?
23. Include or reference technology that provides teachers additional tasks for students?
24. Include teacher guidance for the mindful use of embedded technology to support and enhance student learning?



Try Tool 3

- Examine the extent to which your textbook addresses the overarching areas of concern.



Analysis Tool Components

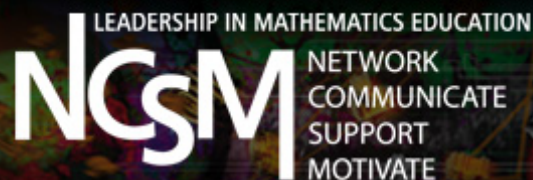
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 - Equity
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 - Technology



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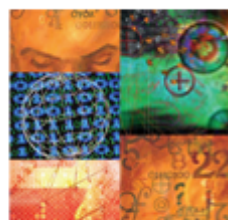
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CCSS Analysis Tools & PD Materials
NCSM Illustrating Mathematical Practices
NCSM Great Task Sample

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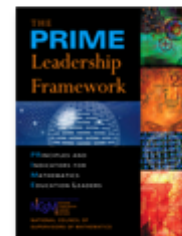
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Tools for Educators

At Inside Mathematics, we've assembled multiple ways for educators to begin to transform their teaching practices. You might be in search of materials and tasks you can use immediately with your students; you can search by grade level and content area below to find core mathematical principles as well as materials developed by the Mathematics Assessment Resource Service (MARS). If you want to develop your understanding of the national Common Core Standards for Mathematical Practice, you can view [connections between the standards and classroom videos](#). If you want to observe exemplar lessons in different content areas and grade levels, visit the [public lessons](#) page. If you are working to enact change in more than one classroom, visit the [tools for coaches](#) and [tools for administrators](#) sections.

TOOLS FOR EDUCATORS

- [Problems of the Month](#)
- [Tools for Coaches](#)
- [Tools for Principals and Administrators](#)

Curriculum, Professional Development, & Administrative Leadership Resources

Tools by Grade

- [Kindergarten Math](#)
- [1st Grade Math](#)
- [2nd Grade Math](#)
- [3rd Grade Math](#)
- [4th Grade Math](#)
- [5th Grade Math](#)
- [6th Grade Math](#)
- [7th Grade Math](#)
- [8th Grade Math](#)
- [Course 1 \(Algebra\)](#)
- [Course 2 \(Geometry\)](#)

Tools by Subject

- [Algebra & Functions](#)
- [Algebraic Properties & Representations](#)
- [Data Analysis](#)
- [Functions & Relations](#)
- [Geometry & Measurement](#)
- [Mathematical Reasoning & Proofs](#)
- [Number Operations](#)
- [Number Properties](#)
- [Patterns, Functions & Algebra](#)
- [Probability](#)
- [Statistics](#)

Mathematics Assessment Project (MAP)

<http://map.mathshell.org.uk/materials>

Mathematics Assessment Project

Shell Center/MARS, University of Nottingham & UC Berkeley



[Home](#) [MAP Overview](#) [Lessons](#) [Tasks](#) [Tests](#) [Standards](#) [Instructions](#) [Log In](#)

Welcome to the Mathematics Assessment Project

On this page

- [Goals](#)
- [Products](#)
- [What's on this site?](#)
- [Who can use the MAP materials?](#)

Important note: *This site, and all the materials here are still in draft or "beta" form: if you find any errors please send feedback to map.feedback@mathshell.org - this includes the software driving this site, so please excuse any bugs and glitches.*

Goals

"And I'm calling on our nation's governors and state education chiefs to develop standards and assessments that don't simply measure whether students can fill in a bubble on a test, but whether they possess 21st Century skills like problem solving and critical thinking and entrepreneurship and creativity."

President Obama, 1 March 2009.

The project is working to design and develop well-engineered assessment tools to support US schools in implementing the [Common Core State Standards](#) for Mathematics (CCSS).

Funding is provided by the Bill and Melinda Gates Foundation through the University of California, Berkeley.

Products

Tools for formative and summative assessment that make knowledge and reasoning visible, and help teachers to guide students in how to improve, and monitor their progress. These tools comprise:

- **Lesson Units for Formative Assessment:** some focused on math concepts, others on non-routine problem solving – 20 per grade for Grades 7-12.
- **Professional Development Modules:** to help teachers with the new pedagogical challenges that formative assessment presents.
- **Summative Assessment Task Collection:** to illustrate the range of performance goals required by CCSS.
- **Prototype Summative Tests:** designed to help teachers and students monitor their progress, these tests provide a model for examinations that may replace or complement current US tests.

The team also contributes to some system capacity building activities within the wider collaboration that the Gates Foundation has assembled, including states and school systems across the US.



Mathematics Assessment Project (MAP)

<http://map.mathshell.org.uk/materials>

- 20 ready-to-use Lesson Units for Formative Assessment for high school. cross referenced to CCSS content and practices standards. (Ultimately 20 per grade 7-12)
- Draft summative assessments, aimed at “College- and Career-Readiness,” presented in two forms:
 - (1) a [Task Collection](#) with each task cross-referenced to the CCSS, and
 - (2) a set of [Prototype Test Forms](#) showing how the tasks might be assembled into balanced assessments.
- Professional development modules are under development



MAP Formative Assessment Lessons

- Assessment task, individual work (15 min)
- Teacher reviews work, creates questions to improve solutions
- (Whole group discussion)
- Partner or small group task to increase understanding, address misconceptions
- Debriefing discussion
- Revision of work on original assessment



The Illustrative Mathematics Project

illustrativemathematics.org

- Hyperlinked CCSS
- Developing a complete set of tasks for each standard
 - Range of difficulty
 - Simple illustrations of single standards to complex tasks spanning many standards.
- Provide a process for submitting, discussing, reviewing, and publishing tasks.
- Launch Team: Phil Daro, William McCallum (chair), Jason Zimba

Tools for the Common Core Standards

commoncoretools.wordpress.com

Tools for the Common Core Standards

News about tools that are being developed to support implementation of the Common Core State Standards



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Progression on Ratios and Proportional Reasoning

Posted on [September 12, 2011](#) by [Bill McCallum](#)

Here is a draft of the [Progression on Ratios and Proportional Reasoning](#). This one took a long time because there is a lot of conflicting and confusing language about ratios and proportional reasoning out in the field, and we struggled with decisions about the extent to which we should try to standardize the language. So comments on this draft would be especially appreciated.

Search

Recent Posts

- [Progression on Ratios and Proportional Reasoning](#)
- [Videos about the standards](#)
- [Essay by Al Cuoco on choosing curriculum aligned to the practices](#)
- [Drafty draft of](#)

illuminations.nctm.org/Lessons.aspx



[Activities](#) | [Lessons](#) | [Standards](#) | [Web Links](#)

NCTM Resources

Lessons

Search Lessons

[View All Lessons](#)

Illustrations has 606 lesson plans available. Select which types of lessons you're looking for, and click **Search**.

Grades

Select All

- ☐
- ☐
- ☐
- ☐

Standards

Select All

- ☐ Number & Operations
- ☐ Algebra
- ☐ Geometry
- ☐ Measurement
- ☐ Data Analysis & Probability

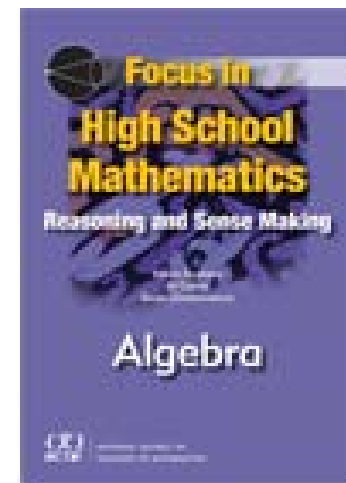
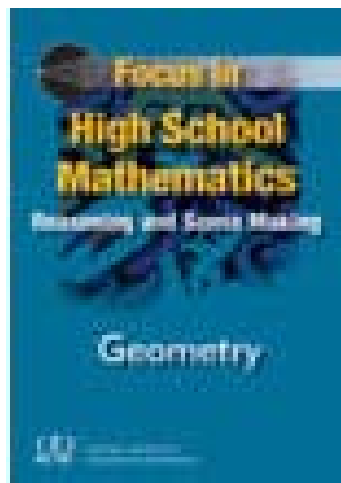
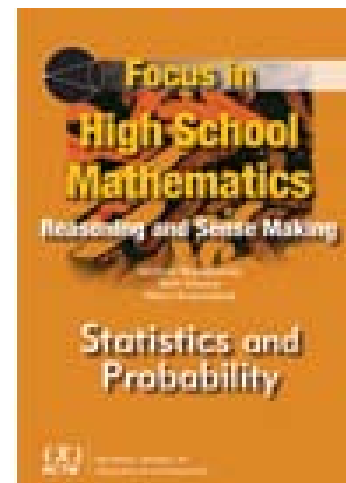
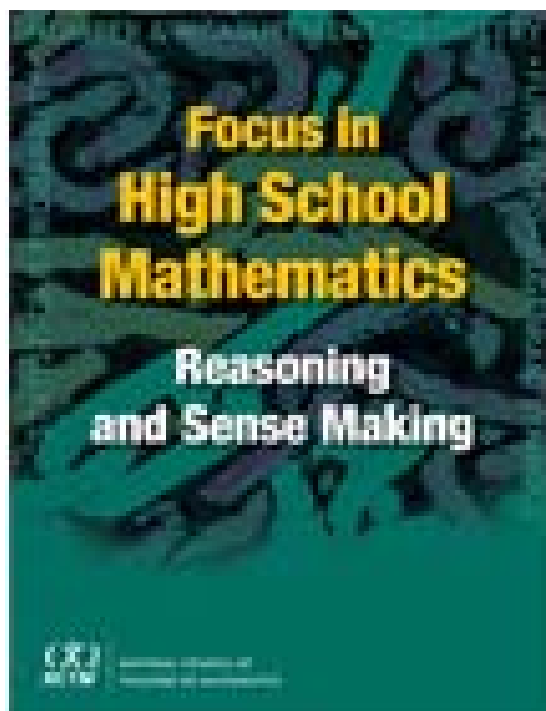
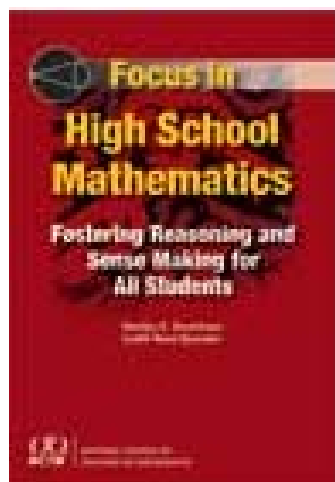
Advanced Options

- ☐ Show only lessons with associated online activities



Search

NCTM's Focus in High School Mathematics: Reasoning and Sense-Making

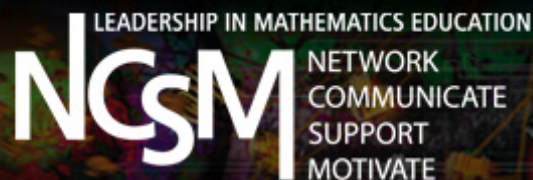




NCSM Resources and Tools

- *Great Tasks* for illustrating the CCSS content and practices standards.
- Webinars:
 - November 8: *CCSS Curriculum Materials Analysis Tools*
 - *Getting Started with the Common Core State Standards: First Steps for Mathematics Education Leaders*
 - *Diving Deeper into the Common Core State Standards*

Mathedleadership.org



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COMMON CORE STATE STANDARDS

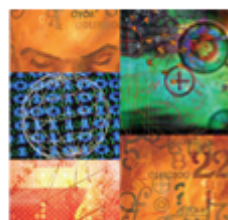
Latest News, including...

2011 Moving Forward Together: Curriculum Assessment and the CCSSM

Resources, including...

Inside Mathematics
CCSS Analysis Tools & PD Materials
NCSM Illustrating Mathematical Practices
NCSM Great Task Sample

Upcoming Event



Fall Leadership Seminars

ATLANTIC CITY, NJ
Oct 19, 2011
ST. LOUIS, MO
Oct 26, 2011
ALBUQUERQUE, NM
Nov 2, 2011

Register Now...



FREE Live Webinar

An Overview of the
CCSS Curriculum
Materials Analysis Tools

November 8, 2011

4:30-5:30PM EST

Register now...

Why I Joined NCSM

"NCSM's PRIME
Leadership Framework
provides actions to reach
goals for excellence."

JEFF LARSON
MIDDLE SCHOOL
PRINCIPAL

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at NCSM...**



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Fall NCSM Events

2011 Leadership Seminars, [register now](#)
2011 NCSM Regional Breakfasts, RSVP via email
to rsvp@explorellearning.com

2012 Annual Conference

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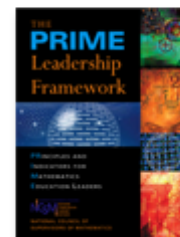
Access Members-Only Resources


...Journals ...Position Papers
...Newsletters ...Podcasts
...and more

THE PRIME Leadership Framework

The ideal resource for
innovative mathematics leaders

Take a Look Inside...





NCSM Professional Development Opportunities

- NCSM Fall One-Day Seminars
 - October 26, 2011, St. Louis
 - November 2, 2011, Albuquerque
- NCSM Webinar:
CCSS Curriculum Materials Analysis Tools
 - November 8, 4:30 -5:30 pm ET
- NCSM Annual Conference
 - April 23-25, 2012, Philadelphia, PA
- NCSM Summer Leadership Academy
 - June 25-27, 2012, Pittsburgh, PA



Reflections

- What next steps will you take to implement CCSS?
- Who will you need to work with?
- What support/information/resources will you need?



Thank You!

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NCSM

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